

Stats 2 - June 2010

- ① $H_0: \mu = 79$
 $H_1: \mu > 79$ (1 tailed test)

From calculator:

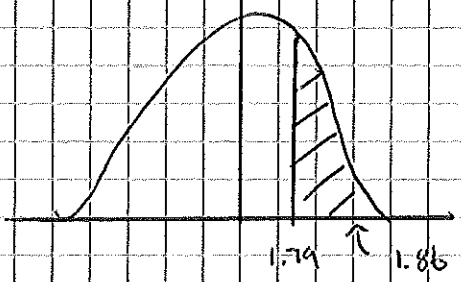
$\sum x = 984$ $\bar{x} = 82$
 $\sum x^2 = 81,030$ $s = 5.5759$
 $n = 12 \rightarrow v = 11$

Don't know σ , and $n < 30$, so use t .

Test statistic: $t = \frac{82 - 79}{\frac{5.5759}{\sqrt{12}}} = 1.86$

Critical value: $t_{(11), 5\%, 1 \text{ tailed}} = 1.796$

Assumption: Number of customers served is normally distributed



$1.86 > 1.796$

\therefore Reject H_0

Evidence at 5% to support belief that number of customers has increased

- ② H_0 : No association between drug & sickness
 H_1 : Association between drug & sickness

2 by 2, so need Yates Correction [degrees of freedom = 1]

Expected	Sick	No sick
Drug	28	52
No Drug	7	13

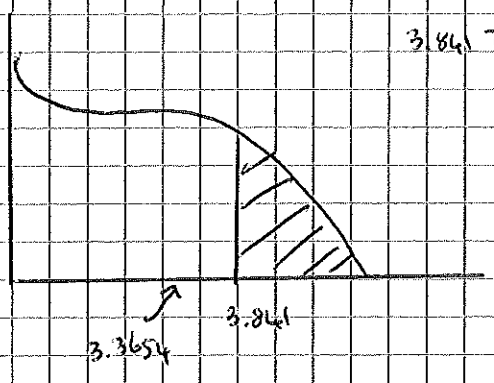
Yates $\frac{[|O - E| - 0.5]^2}{E}$

	sick	No sick
Drug	0.4375	0.2356
No Drug	1.7500	0.9423

Test statistic: $\sum X^2 = 3.3654$

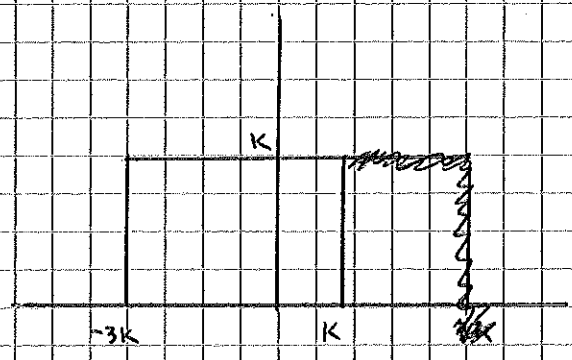
Critical value: 5%, $v = 1 = 3.841$

3.841 > 3.365 : Accept H_0



No evidence at 5% level of an association between drug and prevention of sickness

3) a) i)



ii) Area must = 1

$$\therefore 4k(k) = 1$$

$$\rightarrow 4k^2 = 1$$

$$\rightarrow k^2 = 1/4$$

$$\rightarrow k = 1/2 \text{ as } k > 0$$

b) Rectangular Distribution

$$\therefore E(X) = 1/2 (-3k + k) = 1/2 (-2k) = -k = -1/2$$

$$\text{Var}(X) = 1/2 (k - (-3k))^2$$

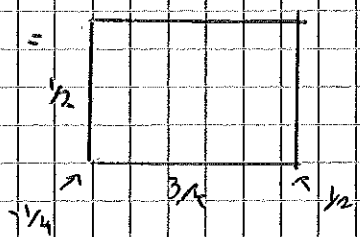
$$= 1/2 (4k)^2 = 1/2 (16k^2) = 8k^2/3 = 1/3$$

$$\rightarrow \frac{4(1/2)^2}{3}$$

$$\rightarrow \text{sd}(X) = \sqrt{1/3} \text{ or } 1/\sqrt{3}$$

c) i) $P(X \geq -1/4)$

From diagram =



$$= 1/2 \times 3/4 = 3/8$$

ii) $P(X \neq -1/4) = 1$

$$\text{as } P(X = -1/4) = 0$$

4)

$$\bar{x} = \frac{0.35}{10} = 0.035$$

$$s^2 = \frac{0.12705}{9} = 0.014116\dots$$

Don't know σ^2 and $n < 30$ so need t

$$n = 10, \nu = 9$$

$t_{(9)}$, 99%, 2 tailed \rightarrow t value of 3.250

$$99\% \text{ CI} = 0.035 \pm 3.25 \times \sqrt{\frac{0.0416}{10}}$$

$$= 0.035 \pm 0.1221$$

$$= (-0.087, 0.157)$$

5) a) i) $X \sim P_0(7)$

i) $P(X \leq 5) = 0.301$ (from tables)

ii) $P(X = 7) = \frac{e^{-7} \times 7^7}{7!} = 0.149$

iii) $P(5 \leq X < 10) = P(X \leq 9) - P(X \leq 4)$
 $= 0.8305 - 0.1730 = 0.6575$

b) $7/8 = 0.875 \rightarrow Y \sim P_0(0.875)$

c) i) 4 engineers \rightarrow 4 available

ii) $Y \sim P_0(0.875)$

$$P(Y < 4) = P(Y=0) + P(Y=1) + P(Y=2) + P(Y=3)$$

$$= e^{-0.875} \times \left[\frac{0.875^0}{0!} + \frac{0.875^1}{1!} + \frac{0.875^2}{2!} + \frac{0.875^3}{3!} \right]$$

$$= 0.4169 \left[1 + 7/8 + 4^2/128 + 3 \times 4^3/3072 \right]$$

$= 0.987760$ = Probability that at least 1 engineer ~~is~~ available has not been called out

\therefore Probability all 4 are Not available $= 1 - 0.987760 = 0.0123$

d) A probably not constant as number of calls per day not likely to be evenly spread out.

6) a) i) $P(R \geq 5) = 0.3 + 0.25 + 0.1 + 0.05 = 0.7$

ii) $E(R) = 3 \times 0.1 + 4 \times 0.2 + 5 \times 0.3 + 6 \times 0.25 + 7 \times 0.1 + 8 \times 0.05 = 5.2$

iii) $E(R^2) = 3^2 \times 0.1 + 4^2 \times 0.2 + 5^2 \times 0.3 + 6^2 \times 0.25 + 7^2 \times 0.1 + 8^2 \times 0.05 = 28.7$

$\therefore \text{Var}(R) = 28.7 - 5.2^2 = 1.66$

b) i) $P(R+S = 6) = 0.1 \times 0.15 = 0.015$

$P(R+S = 7) = 0.1 \times 0.4 = 0.04$
 or $0.2 \times 0.15 = 0.03$ } 0.07

$P(R+S = 8) = 0.1 \times 0.3 = 0.03$
 or $0.2 \times 0.4 = 0.08$
 or $0.3 \times 0.15 = 0.045$ } 0.155

$P(R+S \leq 8) = P(=6) + P(=7) + P(=8) \rightarrow \underline{\underline{0.24}}$

ii) $P(\text{success}) = 0.24$. $P(T \geq 4) = P(T=4) + P(T=5)$

$P(T=4) \rightarrow p = {}^5C_4 \times 0.24^4 \times 0.76^1 = 0.0126$

$P(T=5) \rightarrow p = {}^5C_5 \times 0.24^5 \times 0.76^0 = 0.000796$

$\therefore P(T \geq 4) = \underline{\underline{0.0134}}$

iii) $P(R=4) / (R+S \leq 8) = \frac{P(A \cap B)}{P(B)} = \frac{P(R=4 \text{ and } R+S \leq 8)}{P(R+S \leq 8)}$
 \uparrow \uparrow
 A B
 $= \frac{0.03 + 0.08}{0.24} \leftarrow (4=4)$
 $= \frac{11}{24}$

7) a) Median = 1 [as $P(0 \leq x \leq 1) = 1/2$]

LQ = $1/2$ [as rect distribution]

b) For $0 \leq x \leq 1$, $F(x) = 1/2$

For $1 \leq x \leq 4$, $F(x) = \int f(x)$

(need x as upper limit)

$= \int_1^x 1/18(x-1)^2 dx$

$= \left[1/54(x-1)^3 \right]_1^x$

$= 1/54(x-1)^3 - 1/54(-3)^3$

$= 1/54(x-1)^3 + 1/2$

So $F(x) = 1/2 + 1/54(x-1)^3 + 1/2 = 1 + 1/54(x-1)^3$

$$\begin{aligned}c) P(2 \leq X \leq 3) &= F(3) - F(2) \\ &= 1 + \frac{1}{54}(3-4)^3 - \left[1 + \frac{1}{54}(2-4)^3 \right] \\ &= \frac{5^3}{54} - \frac{4^3}{54} = \frac{7}{54}\end{aligned}$$

b) i) For uq , $F(q) = 0.75$

$$\rightarrow 1 + \frac{1}{54}(x-4)^3 = 0.75$$

$$\rightarrow \frac{1}{54}(x-4)^3 = -0.25$$

$$\rightarrow (x-4)^3 = -13.5 \quad \rightarrow (q-4)^3 = -13.5$$

ii)

$$\rightarrow x-4 = \sqrt[3]{-13.5}$$

$$\begin{aligned}\rightarrow x &= \sqrt[3]{-13.5} + 4 \\ &= 1.619 \text{ (3dp)}\end{aligned}$$